



Adaptive shape prior in graph cut image segmentation

Hui Wang*, Hong Zhang, Nilanjan Ray

Department of Computing Science, University of Alberta, 2-21 Athabasca Hall, Edmonton, Alberta, Canada T6G 2E8

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ABSTRACT

This paper presents a novel method to apply shape priors adaptively in graph cut image segmentation. By incorporating shape priors adaptively, we provide a flexible way to impose the shape priors selectively at pixels where image labels are difficult to determine during the graph cut segmentation. This is in contrast to the use of shape priors indiscriminately at all pixels in existing image segmentation approaches, which may fail if the parameters for the shape prior term are not chosen appropriately. We integrate the proposed method in two existing graph cut image segmentation algorithms, one with shape template and the other with the star shape prior. To determine the need for a shape prior at each pixel, our experiments make use of either the original image or an enhanced version of the original image by smoothing. Experimental results in multiple application domains demonstrate the generality and superior performance of our adaptive shape prior method.

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1. Introduction

Image segmentation has always been an important and challenging task in computer vision. Since Boykov and Jolly [1] introduced the application of the graph cut algorithm into image segmentation, graph cut has become one of the leading approaches in image segmentation in the last decade, because it not only allows one to incorporate user interaction, but also is an efficient and globally optimal algorithm.

More recently, in order to handle noisy images or images with object occlusions effectively, new methods have been developed to exploit shape priors. Freedman and Zhang proposed to incorporate shape priors by matching the segmented curve with a shape template [2]. Veksler showed how to implement a shape prior for objects defined as star shaped [3]. Das et al. presented a similar idea to incorporate shape priors for shapes defined as compact [4]. In addition, some research activities focus on one or two particular types of objects with particular shapes [5,6], some on incorporating multiple shape priors into one image [7], and yet some on shape representation and general shape constraints [8,9].

One of the problematic issues of the graph cut framework is the selection of weights on the various terms in the energy function. These weights are usually tuned beforehand by the developer of the algorithm to achieve the best result for a certain type of images [10]. For example, Peng and Veksler [10] designed a parameter selection method by measuring segmentation quality

based on different features of the segmentation. They ran graph cut for different parameter values and chose the parameters which produce segmentation of the highest quality. However, their method only targets issues of selecting the parameters between the data term and the boundary term in the energy function, while setting a constant weight on the shape prior. For images corrupted by significant noise and intensity inhomogeneities, the needs for a shape prior at different pixels are different in general. Therefore, setting a constant weight on the shape prior term for all pixels may not be appropriate. As an example, columns (b) to (d) in Figs. 1 and 2 show examples where different parameter settings for the shape prior can lead to very different segmentation results.

To solve the issue described above, we propose to impose shape constraints selectively, by applying the shape prior adaptively in graph cut. To determine the need for the shape prior at each pixel, we derive a shape weight term based on image intensity. The intuition behind this is that if a pair of neighboring pixels has a small difference in appearance, there should be a higher weight for the shape constraint in the energy function to compensate for the weak or missing edge information. In this way our method gives flexibility in applying a shape prior, and helps obtain a segmentation result that matches better with the shape prior. As will be seen in our paper, this weight on the shape constraint can be easily calculated without much additional computational cost.

An adaptive graph cut idea has been proposed by Song et al. [11] where they proposed a framework for segmentation of brain tumors in MRI images within an iterative scheme. They incorporated a shape atlas of adaptive probabilistic priors into the graph cut energy function by combining it with the image intensity

* Corresponding author. Tel.: +1 780 492 7188; fax: +1 780 492 1071.

E-mail addresses: wanghui@ualberta.net (H. Wang), hzhang@ualberta.ca (H. Zhang).

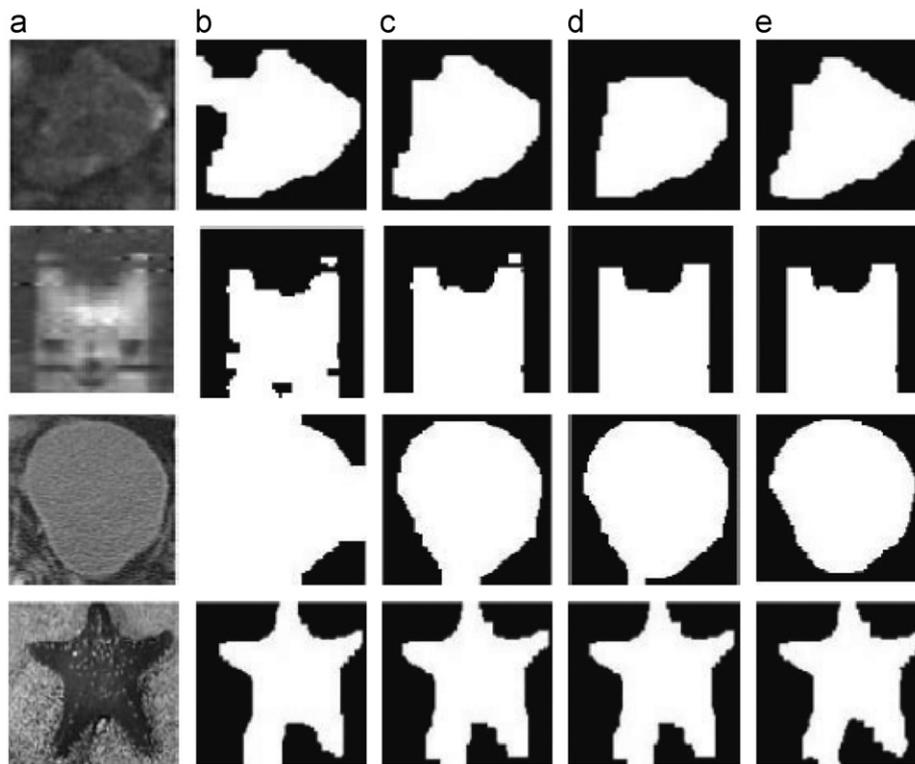


Fig. 1. Results from the shape template based method [2]. Column (a) shows the original images. Columns (b)–(d) show segmentation results from Freeman and Zhang's original shape template method with $\lambda = 0.2, 0.5$ and 0.8 . Column (e) shows segmentation results from our adaptive shape prior applied to Freedman and Zhang's shape prior method. Column (e) uses smoothed images as the probability maps α .

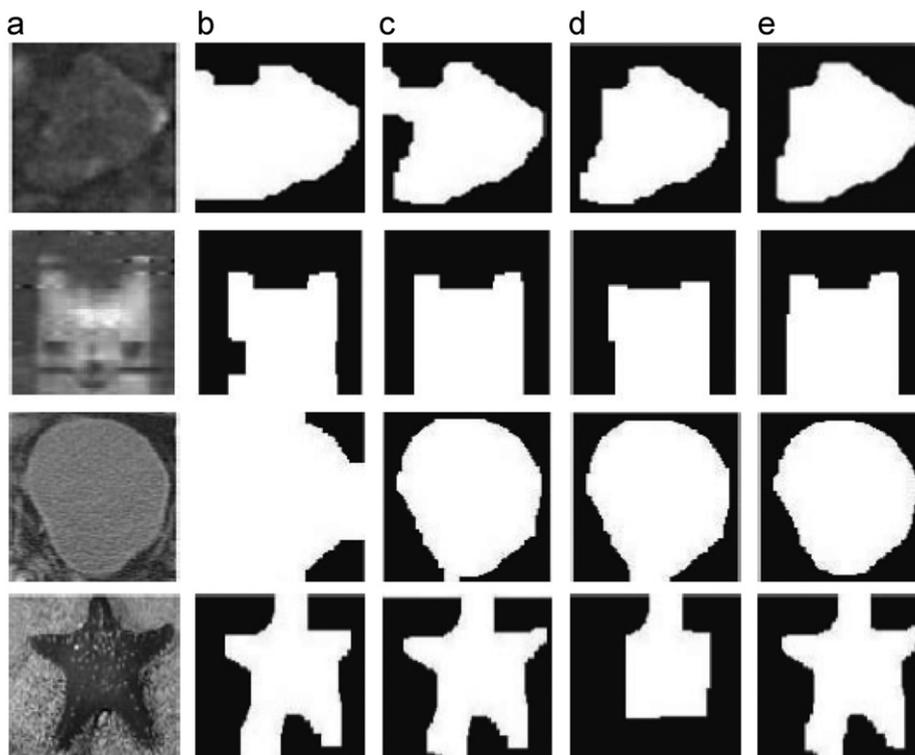


Fig. 2. Results from the star shape prior method [3]. Column (a) shows the original images. Columns (b)–(d) show segmentation results from the original star shape prior method with $\lambda = 0.2, 0.5$ and 0.8 . Column (e) shows segmentation results from our adaptive shape prior applied to Veksler's star shape prior method. Column (e) uses smoothed images as the probability maps α .

distribution. However, the adaptive idea suggested in [11] works on the combined data term only, instead of on the pairwise terms. Furthermore, the performance of their method relies a lot on the

accuracy of the atlas, and several parameters, such as the weight λ between the data and boundary terms, as well as the scale for calculating the neighborhood links.

In another study, Bar-Yosef et al. [12] proposed a variational method for model based segmentation with an adaptive shape prior, with the help of a shape confidence map. Their prior confidence map was defined to select a shape model among many shape models, with the maximum confidence at each pixel to reflect the reliability of the shape prior at each pixel. The prior confidence map then determines if the segmentation method should follow the shape prior or not at each pixel location. However, their method only focuses on the variational framework with many shape prior models, and has only been applied in one specific application. In contrast to the previous work, our proposed method tackles adaptive shape priors from a different angle. The weight on the shape constraint is obtained from the available image level information, and it reflects how much each pair of pixels needs the shape prior to help with image segmentation. In other words, we measure the need for the shape prior between a pair of pixels, instead of the reliability of the shape prior.

The rest of the paper is organized as follows. In Section 2 we present the background for the graph cut segmentation with shape priors in a unified way to combine the boundary and shape term in the energy function. In Section 3 we first describe the issue of parameter selection on the relative importance of each term in graph cut, then present our proposed method for adaptive shape priors and give examples of applying it in some existing graph cut methods with shape priors. In Section 4 we provide the experimental results to demonstrate the generality and superior performance of our approach. Finally, in Section 5 we state conclusions and future work.

2. Background

2.1. Graph cut image segmentation

Many segmentation problems can be formulated in terms of energy minimization. Such energy minimization problems can be further formulated into a maximum flow problem in a graph. Under most formulations of such problems, the minimum energy solution corresponds to the maximum a posteriori estimate of a solution. The term graph cut is applied specifically to those models which employ a max-flow/min-cut optimization [1]. The details on the graph cut energy function are presented in this section.

2.1.1. Graph cut energy function

Graph cut segmentation achieves an optimal solution by minimizing an energy function via the max-flow min-cut algorithm [1]

$$E(A) = \mu D(A) + B(A) \tag{1}$$

where $A = (A_1, \dots, A_p, \dots, A_{|P|})$ represents a binary vector whose component A_p specifies the assignment of background or foreground to pixel p in an arbitrary set of data elements P in an image I , and μ is a non-negative coefficient which specifies a relative importance between the boundary term B and the region term D [1].

To be more specific, Eq. (1) is usually written as

$$E = \mu \sum_{p \in P} D_p(f_p) + \sum_{(p,q) \in N, f_p \neq f_q} B_{pq}(f_p, f_q) \tag{2}$$

where N is the set of neighboring pixels, and f_p represents label assigned to pixel p . The particular forms for $D_p(f_p)$ and $B_{pq}(f_p, f_q)$ are discussed in the following sections.

2.1.2. Region term

The region term D_p assumes that the penalties for assigning pixel p to “foreground” or “background” are given. One example of defining the region term D_p is to apply the negative log-likelihood model, which is originally motivated by the MAP-Markov Random Field formulation [1].

$$D_p(\text{“obj”}) = -\ln \Pr(I_p | \text{“obj”}) \tag{3}$$

$$D_p(\text{“bkg”}) = -\ln \Pr(I_p | \text{“bkg”}) \tag{4}$$

where I_p represents the image intensity of pixel p , “obj” and “bkg” represent object and background respectively.

2.1.3. Boundary term

In graph cut, the boundary term B_{pq} is a penalty term for the discontinuity between a pixel pair p and q . For graph cut without shape prior, $B_{pq} = V_{pq}$, where

$$V_{pq} = e^{-(I_p - I_q)^2 / 2\sigma^2} \cdot \frac{1}{\text{dist}(p,q)} \tag{5}$$

σ is a constant and $\text{dist}(p,q)$ is usually calculated as the Euclidean distance between pixels p and q [1].

2.2. Shape priors in graph cut

If an object of a certain shape is expected as the output of a segmentation algorithm, a shape prior can be used to impose a constraint on the shape of the foreground region. A number of graph cut methods incorporate shape priors by modifying the pairwise term B in the following way [2–5,8,9]:

$$B_{pq} = V_{pq} + \lambda V'_{pq} \tag{6}$$

where V'_{pq} represents the newly added shape prior term, and λ is a constant which measures the relative importance of the shape constraint. V'_{pq} is defined in various ways depending on the type of the shape prior, to penalize the discrepancy between the segmented shape and the expected shape.

Some methods express shape constraint using a shape template such as ellipse [13,14] or circle [13], while other irregular shape templates are also possible [2,6]. More recently, the shape constraints in terms of a class of shapes such as the star shape [3] and compact shape [4] have also been proposed. These shape priors are more general and flexible than a single shape template. In all cases, a shape prior needs to be chosen in such a way that is possible to be expressed in a graph representable form. Some specific forms of V'_{pq} in two existing methods will be discussed in the next section.

3. Proposed method

In this section, we will first describe the issue of parameter selection on the relative importance of each term in graph cut, present our proposed method for applying a shape prior adaptively, and then we will show how we can apply our method in two existing graph cut algorithms that use a shape prior.

3.1. Parameter selection for shape priors in graph cut

One of the outstanding issues of the graph cut framework is the selection of weights on various terms in the energy function. These weights are usually tuned beforehand by the developer of the algorithm to achieve the best result for a certain type of images [10]. Several papers on graph cut with shape priors have mentioned the need for a user to adjust λ depending on the type of image. However, for images corrupted by significant noise and

intensity inhomogeneities, the needs of a shape prior at different pixels might vary significantly. In other words, setting a constant value λ for all pixels on the whole image is not appropriate. Again, we refer to Figs. 1 and 2 to demonstrate the sensitivity of the segmentation result to the choice of λ .

As mentioned, Peng and Veksler [10] studied a parameter selection method for μ by measuring segmentation quality. Further to the selection of μ , our proposed method solves the problem of choosing λ on the shape constraint V'_{pq} adaptively on a per pixel basis. In other words, our method uses a spatially varying weight on the shape prior.

3.2. Adaptive shape prior

Based on our discussion in the previous section, we propose to incorporate a shape prior adaptively, according to the needs of different pixels. Specifically, we notice that the pairwise term in (6) can be modified in the following way that replaces the constant λ with an adaptive weight S_{pq} :

$$B_{pq} = V_{pq} + S_{pq}V'_{pq} \quad (7)$$

The weight S_{pq} can be estimated from either the original image or an enhanced version of it that we refer to as probability map α , which will be discussed further in the next section. Intuitively, the addition of S_{pq} allows us to impose a stronger shape prior term at locations where the edge information is weaker or less obvious, and vice versa.

3.3. Shape weight S_{pq}

The shape weight S_{pq} can be defined in various ways. The higher the similarity is between pixels p and q , the less information there is in the image to allow graph cut to find the boundary of the object, and the stronger the shape constraint should be to help a graph cut algorithm.

First, we introduce a probability map α which has the same size as the image to be segmented. We let α_p denote the likelihood of pixel p belonging to the foreground. The value of α_p should be between 0 and 1. Then we define S_{pq} in terms of α . To reflect the difference in the probability value between pixels p and q , we use $S_{pq} = e^{-(\alpha_p - \alpha_q)^2}$ in our experiments.

3.4. Probability map α

Various methods exist to compute the probability map α which reflects the likelihood of each pixel belonging to the foreground object. For example, α could be obtained from supervised learning techniques [15]. In fact, in the simplest case, α can be a smoothed version of the original image.

In our experiments, we will show image segmentation results by using a smoothed version of the original images. Since the focus of this paper is not to compare the performance of unsupervised and supervised methods for computing α , we only discuss experiments that use smoothing. In our experiments, the smoothed images are generated from applying Gaussian filter on the original images. To demonstrate the generality of our method, our experiments include images in four different applications: (a) ore images in mining, (b) shovel tooth images from an excavation shovel, (c) bladder images in medical applications and (d) star fish images.

3.5. Adaptive shape template method

Freedman and Zhang introduced the idea of incorporating a shape template in the form of level set to graph cut [2]. Their method begins with the assumption that the shape prior is a

single fixed template [2]. In order to incorporate the shape prior, they modified the original energy function of graph cut by the shape term in Eq. (6) as

$$V'_{pq} = \bar{\phi} \left(\frac{p+q}{2} \right) \quad (8)$$

where $\bar{\phi}$ is a regular unsigned distance function whose zero level set corresponds to the shape template curve \bar{c} . By adding this shape energy term V'_{pq} , minimization of the graph cut energy function encourages the object boundary to be aligned with the zero level set [2].

To be more detailed, after adding the shape energy term V'_{pq} , the energy function for Freeman and Zhang's shape template method can be written as

$$E(f) = \mu \sum_{p \in P} D_p(f_p) + \sum_{(p,q) \in N: f_p \neq f_q} V_{pq}(f_p, f_q) + \lambda \sum_{(p,q) \in N: f_p \neq f_q} \bar{\phi} \left(\frac{p+q}{2} \right) \quad (9)$$

A drawback of Freeman and Zhang's method is the requirement of object-template alignment through a variety of transformations which are computationally expensive. Another more relevant limitation is the difficulty in choosing a proper λ , which is a common problem for existing graph cut methods with shape priors.

Following our proposed adaptive shape prior pairwise term, we can easily replace the pairwise terms in the energy function (9) with the new pairwise term B_{pq} . Therefore, energy function (9) for Freedman and Zhang's method becomes

$$E(f) = \mu \sum_{p \in P} D_p(f_p) + \sum_{(p,q) \in N: f_p \neq f_q} V_{pq}(f_p, f_q) + \sum_{(p,q) \in N: f_p \neq f_q} S_{pq} \bar{\phi} \left(\frac{p+q}{2} \right) \quad (10)$$

3.6. Adaptive star shape method

A recent graph cut method with a generic shape prior is the star shape prior method [3]. The star shape prior is not specific to any particular shape, but rather defines a class of shapes. With the assumption that the center of the object is known, the star shape prior method adds a shape constraint to the graph cut energy function as described below.

Consider the center of the star shape is denoted as c . Let 1 and 0 be the object label and the background labels, respectively. To get an object segment of a star shape, for any point p inside the object, we have to insure that every single point q on a straight line connecting c and p is also inside the object. This implies that if p is assigned label 1, then every point between point c and p is also assigned 1. With the assumption that q is between c and p , the star shape method defines the following pairwise shape constraint term V'_{pq} :

$$V'_{pq}(f_p, f_q) = \begin{cases} 0 & \text{if } f_p = f_q, \\ \infty & \text{if } f_p = 1 \text{ and } f_q = 0, \\ \beta & \text{if } f_p = 0 \text{ and } f_q = 1 \end{cases} \quad (11)$$

where β is a weight constant.

To be more detailed, after adding the shape energy term V'_{pq} , the energy function for the star shape method can be written as

$$E(f) = \mu \sum_{p \in P} D_p(f_p) + \sum_{(p,q) \in N: f_p \neq f_q} V_{pq}(f_p, f_q) + \lambda \sum_{(p,q) \in N: f_p \neq f_q} V'_{pq}(f_p, f_q) \quad (12)$$

where V'_{pq} is a shape energy term defined by (11).

Similar to other shape prior methods in graph cut segmentation, the star shape method still has the limitation with parameter selection for different terms in the energy function. As discussed in [3], β needs to be chosen appropriately by the user in order to obtain good segmentation results. It is obvious that λ and β are relative weights, and the problem of selecting them remains.

Following similar procedure as for the shape template method, our proposed adaptive shape prior pairwise terms can easily replace the pairwise terms B_{pq} in the energy function (12). Therefore energy function (12) becomes

$$E(f) = \mu \sum_{p \in P} D_p(f_p) + \sum_{(p,q) \in N_{f_p} \neq f_q} V_{pq}(f_p, f_q) + \sum_{(p,q) \in N_{f_p} \neq f_q} S_{pq} V'_{pq}(f_p, f_q) \quad (13)$$

3.7. Optimality of the new energy function

In Eqs. (10) and (13), the V_{pq} terms are defined the same as in (5). The difference lies in the shape term V'_{pq} . According to [16], an energy function can be optimized exactly with a graph cut if all the pairwise terms are submodular, that is, a binary function g of two variables is submodular if $g(0,0) + g(1,1) \leq g(1,0) + g(0,1)$.

For both Eqs. (10) and (13), we have $V(0,0) = 0$ and $V(1,1) = 0$. As well we have $V'(0,0) = 0$ and $V'(1,1) = 0$. So V_{pq} and V'_{pq} are clearly submodular. It is also easy to prove that $S_{pq} V'_{pq}$ is also submodular. Therefore, we can construct a graph according to [16] and obtain optimized solutions (10) and (13) via max-flow/min-cut algorithm.

4. Experimental results

To validate our proposed shape prior method, we have run experiments on two graph cut methods with shape priors, detailed in Section 3. We use a MATLAB wrapper with the C++ maxflow code by Boykov and Kolmogorov [17]. When comparing our proposed method to Freedman and Zhang's shape template method [2], the shape template is introduced in the same way, i.e., we assume an aligned template as the shape prior. The aligned shape templates for each image are exactly the same for both Freedman and Zhang's method and our method, and are done manually in all our experiments. As mentioned in [2], the key assumption of Freeman and Zhang's method is that, based on the user input, the shape template can be pretty well aligned with the image using the Procrustes Method [18]. Given the aligned template, the distance function can be easily computed via scaling, as the input to the graph cut energy function. It is also mentioned that the rigid transformation computed via the Procrustes Method will not be extremely accurate, however the algorithm is robust to the situation in which the template is not exact.

When comparing our method to the original star shape prior method [3], we also perform our experiments based on exactly the same user initialization to specify the center of the object to be segmented. Therefore, for each image, the user specifies the center of the object, and exactly the same center will be the initialization for both our method and the original star shape method.

We perform our experiments by utilizing a smoothed version of the original image as the probability map α . The smoothed images are generated from applying Gaussian filter on the original images. After obtaining the probability map α , the probability map is combined with the shape prior from either Freedman's method or star shape prior method, and finally the corresponding energy function is minimized via graph cut.

Figs. 1 and 2 show the comparison results of our method to the shape template method [2] and the star shape prior method [3],

Table 1

Statistical results comparing shape template method. Results with adaptive shape prior (ASP) method by using the enhanced filtered images as S_{pq} are shown in the last column. Very similar results were obtained by applying other types of probability maps as S_{pq} .

Shape template method		$\lambda = 0.2$	$\lambda = 0.5$	$\lambda = 0.8$	ASP using denoised image as S_{pq}
Oil sard images	Accuracy	0.95	0.97	0.98	0.98
	Jaccard	0.39	0.95	0.95	0.96
Tooth images	Accuracy	0.32	0.36	0.83	0.92
	Jaccard	0.31	0.81	0.85	0.93
Bladder images	Accuracy	0.33	0.84	0.85	0.85
	Jaccard	0.78	0.79	0.78	0.79
Starfish images	Accuracy	0.32	0.83	0.83	0.84
	Jaccard	0.77	0.73	0.79	0.80

Table 2

Statistical results comparing star shape method. Results with adaptive shape prior (ASP) method by using the enhanced filtered images as S_{pq} are shown in the last column. Very similar results were obtained by applying other types of probability maps as S_{pq} .

Star shape method		$\lambda = 0.2$	$\lambda = 0.5$	$\lambda = 0.8$	ASP using denoised image as S_{pq}
Oil sand images	Accuracy	0.96	0.36	0.96	0.97
	Jaccard	0.92	0.92	0.92	0.92
Tooth images	Accuracy	0.72	0.81	0.80	0.91
	Jaccard	0.55	0.65	0.69	0.70
Bladder images	Accuracy	0.34	0.85	0.84	0.85
	Jaccard	0.78	0.79	0.79	0.79
Starfish images	Accuracy	0.84	0.87	0.86	0.86
	Jaccard	0.61	0.62	0.63	0.63

respectively. In both figures, the original images are shown in column (a), and results obtained by the competing methods are shown in columns (b) to (d) with different values of λ . Our results are shown in column (e) by using a smoothed image of the original image as the probability map.

Tables 1 and 2 show the statistical results. To evaluate the performance of the algorithms quantitatively, we apply two popular evaluation metrics: Jaccard index [19] and pixel accuracy. Define TP , TN , FP , FN as the number of pixels being labeled as true positive, true negative, false positive, false negative, respectively. Jaccard index is defined as $TP/(TP+FP+FN)$, and it is the ratio of intersection and union of the segmented region and ground truth region. On the other hand, pixel accuracy is defined as $(TP+TN)/(TP+TN+FP+FN)$, and it incorporates both correctly labeled foreground and background pixels.

In total, our experiments included 20 oil sand images, 46 tooth images, 15 bladder images and 10 starfish images. The last two columns in the tables demonstrate the superior performance of our method over the competing methods. The highlight columns show the best performance in each row. It is clear that our method obtains better segmentation results most of the time without the need to optimize with regard to λ , while the two competing methods both need to tune the parameter λ .

We obtain almost the same segmentation results even though we use different techniques for obtaining S_{pq} . This shows that our method is flexible in terms of how α is generated. Since we are not interested in picking the best method for computing α , comparison among various types of α is not further examined in our studies.

5. Conclusions and future work

We have proposed an adaptive method for incorporating shape priors into graph cut to eliminate the need to tune the weight of

the shape constraint. We have shown that the proposed method can be easily applied to various types of graph cut methods with shape priors, such as Freedman and Zhang's method with a shape template [2], and Veskler's graph cut method with the star shape prior [3].

In addition, we have validated our method for various types of images and obtained better results than the state of the art graph cut methods with shape priors. Although there is a preprocessing step to obtain a probability map α in our method, this step is equivalent to image enhancement, and it is therefore straight forward, automatic and with little extra computational cost. With different ways of obtaining the probability map, the segmentation results are consistently better than those using the other two existing shape prior methods.

Similar to most existing graph cut based segmentation method, the performance of our method still depends on the parameter μ , the weight on the data term D . So as in all graph cut based methods, we have to obtain a proper μ for the images in our experiments. Also, similar to most shape template based methods, our method needs to align the shape template to obtain a proper segmentation result.

One direction of future work is to see if our adaptive shape prior idea can be extended to segmentation algorithms other than graph cut. Another direction for future research is to examine whether more types of shapes can be incorporated into the graph cut framework. While we have seen that the current method is relatively robust compared to other existing shape prior methods, it will be interesting to see whether this robustness holds in the case of greater variations in shapes and images, and if not, how the algorithm may be modified to account for these changes.

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Hui Wang received her Ph.D. degree from the Department of Computing Science at the University of Alberta, Canada. She received her bachelor degree in Computer Science from Central South University, China, in 2004 and master's degree in Electronic Engineering from University of Regina, Canada, in 2006. Her current research interests include image analysis, computer vision and machine learning.

Hong Zhang received his B.Sc. degree from Northeastern University, Boston, USA, and his Ph.D. degree from Purdue University, West Lafayette (IN), USA, both in Electrical and Computer Engineering. He is currently a Professor in the Department of Computing Science, University of Alberta, and Director of the Centre for Intelligent Mining Systems. His current research interests include robotics, computer vision, image processing, and intelligent systems.

Nilanjan Ray received his bachelor degree in mechanical engineering from Jadavpur University, Calcutta, India, in 1995, masters degree in computer science from the Indian Statistical Institute, Calcutta, in 1997, and Ph.D. in electrical engineering from the University of Virginia, Charlottesville, in May, 2003. After having two years of postdoctoral research and a year of industrial work experience Nilanjan joined the department of Computing Science, University of Alberta in July 2006. His current research area is image analysis.